Fixed point indices of iterates of orientation-reversing homeomorphisms

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Let $f\colon \mathbb{R}^m \to \mathbb{R}^m$ be a self-homeomorphism and let p be an isolated fixed point for each iterate of f. Therefore, the fixed point index, $\operatorname{ind}(f^n, p)$, is a well-defined integer for each n, and thus it is possible to consider a whole sequence of indices $\{\operatorname{ind}(f^n, p)\}_{n=1}^{\infty}$. It is known (cf. [2]) that under the assumption that $\{p\}$ is an isolated invariant set, the possible forms of indices in the class of orientation-reversing self-homeomorphisms of $\mathbb{R}^3$ are very restricted. In this talk, first, we present the complete solution to Problem 10.2 in [1], and secondly, rejecting the assumption that $\{p\}$ is necessarily an isolated invariant set, we give a full characterization of $\{\operatorname{ind}(f^n, p)\}_{n=1}^{\infty}$ in the class of orientation-reversing self-homeomorphisms of $\mathbb{R}^m$. This is a joint work with Prof. Graff.

References

[1] H. Barge, K. Wójcik, *Mayer–Vietrois property of the fixed point index*, Topol. Methods Nonlinear Anal. **50** (2017), 643-667.

[2] L. Hernández-Corbato, P. Le Calvez, F. R. Ruiz del Portal, *About the homological discrete Conley index of isolated invariant acyclic continua*, Geometry & Topology **17** (2013), 2977-3026.