A priori regularity estimates for equations degenerating on nodal sets

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This talk is based on a joint works with Giorgio Tortone[2] and Stefano Vita[1]
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We prove *a priori* and *a posteriori* Hölder bounds and Schauder $C^{1,\alpha}$ estimates for continuous solutions to singular/degenerate equations with variable coefficients of type

$$\operatorname{div}(|u|^a A\nabla w) = 0 \qquad \text{in } \Omega \subset \mathbf{R^n},$$

where the weight u solves an elliptic equation of type $\operatorname{div}(A\nabla u) = 0$ with a Lipschitz-continuous and uniformly elliptic matrix A and has a nontrivial, possibly singular, nodal set.

Such estimates are uniform with respect to u in a class of normalized solutions having bounded Almgren's frequency. More precisely, we provide *a priori* Hölder bounds in any space dimension, and Schauder estimates when $n = 2$. When $a = 2$, the results apply to the ratios of two solutions to the same PDE sharing their zero sets. Then, one can infer higher order boundary Harnack principles on nodal domains by applying the Schauder estimates for solutions to the auxiliary degenerate equation. The results are based upon a fine blow-up argument, Liouville theorems and quasiconformal maps.

References

[1] S. Terracini, G. Tortone and S. Vita, *Higher order boundary Harnack principle on nodal domains via degenerate equations*, to appear on Arch. Rat. Mech. Anal .

[2] S. Terracini, G. Tortone and S. Vita, A priori regularity estimates for equations degenerating on nodal sets, preprint 2024