Normalised solutions to a Schrödinger equation with potential

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In [1] we consider a Schrödinger type equation of the form

$$-\Delta u + (\lambda + V(x))u = |u|^{p-2}u \tag{1}$$

in $\mathbb{R}^N$, with a non radial potential V under the mass constraint

$$\int_{\mathbb{R}^N} v^2 = \rho^2. \tag{2}$$

We provide some sufficient conditions about V for existence of solutions $(u, \lambda) \in H^1(\mathbb{R}^N) \times (0, \infty)$ for powers $2 + \frac{4}{N} < p < \frac{2N}{N-2}$. The potential is allowed to have singularities. λ appears as a Lagrange multiplier, due to the mass constraint (2). The proof is variational, based on a min-max argument.

References

[1] T. Bartsch, R. Molle, M. Rizzi, G. Verzini, Normalized solutions of mass supercritical Schrödinger equations with potential. (English summary) Comm. Partial Differential Equations **46** (2021), no. 9, 1729–1756.