Schrödinger equation in dimension two with competing logarithmic self-interaction

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The aim of this talk is to deal with a scalar field equation in dimension 2 in presence of two logarithmic nonlocal terms in competition, namely

$$-\Delta u + (\log|\cdot| * |u|^2)u = (\log|\cdot| * |u|^q)|u|^{q-2}u, \qquad \text{in } \mathbb{R}^2, \qquad (\mathcal{P})$$

with $q > 2$.

Formally, solutions of $(\mathcal{P})$ can be found as critical points of the functional

$$I(u) = \int_{\mathbb{R}^2} |\nabla u|^2\,dx + \frac{1}{2}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(|x-y|)|u(x)|^2|u(y)|^2\,dx\,dy - \frac{1}{q}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(|x-y|)|u(x)|^q|u(y)|^q\,dx\,dy.$$

However, both the two nonlocal terms have not a fixed sign, are not well defined in $H^1(\mathbb{R}^2)$ and are in competition. This requires, as first step, a careful study of suitable weighted Sobolev spaces, with coercive potentials, and their embedding properties.

The results have been obtained in joint works with A. Azzollini, P. d'Avenia, S. Secchi.