Normalized solutions to a Choquard equation involving mixed local and nonlocal operators

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We study the existence of normalized solutions for the following Choquard type equation involving mixed diffusion type operators:

$$\begin{array}{rcl} \mathcal{L}u + u & = & \mu(I_\alpha * |u|^p)|u|^{p-2}u \ \text{ in } \mathbb{R}^n, \\ \int_{\mathbb{R}^n} |u|^2 dx & = & \tau, \end{array}$$

where $\frac{n+\alpha}{n} \leq p \leq \frac{2s+n+\alpha}{n}$, $\tau > 0$ is a constant, $\mu > 0$ is a parameter, I_α is the Riesz potential of order $\alpha \in (0, n)$ defined by

$$I_\alpha = \frac{A_{n,\alpha}}{|x|^{n-\alpha}}, \ \text{ with } A_{n,\alpha} = \frac{\Gamma(\frac{n-\alpha}{2})}{\pi^{\frac{n}{2}} 2^\alpha \Gamma(\frac{\alpha}{2})} \ \text{ for every } x \in \mathbb{R}^n \setminus \{0\}$$

and the mixed operator $\mathcal{L}$ is given by

$$\mathcal{L} = -\Delta + \lambda(-\Delta)^s \ \text{ for some } s \in (0,1) \text{ and parameter } \lambda > 0.$$

We also provide regularity results of these solutions. Further, the equivalence between existence of normalized solutions and the existence of normalized ground states is established.