Existence and the Number of Travelling Wave Solutions of the Beam Equation with Jumping Nonlinearity

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We focus on the travelling wave solutions of the boundary value problem for fourth–order partial differential equation

$$u_{tt} + u_{xxxx} + au^+ - bu^- + g(u) = 1, \quad x \in \mathbb{R},\, t > 0, \tag{1}$$

where a, $b > 0$, $u^\pm = \max\{\pm u, 0\}$ and $g(1/a) = 0$. Such problems are usually used as a model of an asymmetrically supported bending beam or as a generalised model of a suspension bridge.

First, we deal with the existence of the solution under significantly weakened assumptions compared to those previously used in literature. Using variational methods, in particular Mountain Pass Theorem together with a nonzero weak convergence after a suitable translation, we show that there are infinitely many homoclinic travelling wave solutions of the equation (1) with arbitrary wave speed from the interval $\left(c^*, \sqrt[4]{4a}\right)$, where $c^* > \sqrt[4]{4b}$. This limitation on the lower bound is due to the term bu^- in the nonlinearity.

Further, we present a special form of classical solutions of (1) which help us answer some open questions about number of travelling waves with the same fixed wave speed from the interval mentioned above.

References

[1] Holubová, G., Levá, H. *Travelling wave solutions of the beam equation with jumping nonlinearity.* Journal of Mathematical Analysis and Applications, 2023, 527.2: 127466.

[2] Holubová, G., Levá, H., Nečesal, P. *Lower bounds for admissible values of the travelling wave speed in asymmetrically supported beam.* Submitted, 2024.