Existence and multiplicity of solutions for a concave-convex system of ODEs

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In this lecture we discuss the system of ODEs

$$\begin{cases} -u'' = \lambda v^r + v^p & \text{in } (0,1), \\ -v'' = u^q & \text{in } (0,1), \\ u > 0,\ v > 0 & \text{in } (0,1), \\ u(0) = u(1) = 0, \\ v(0) = v(1) = 0, \end{cases} \tag{1}$$

where $\lambda > 0$, $r \in (0,1)$, $p \in (1,\infty)$, and q is such that $qr < 1$.

System (1) has been recently studied in higher dimensions by dos Santos, and Agudelo et al., motivated by the results of Ambrosetti, Brezis and Cerami from 1994. We explore the one dimensional case, which has not been treated yet and which usually renders more precise information about the solutions.

We discuss some results related to regularity, existence, and multiplicity of solutions of (1). We also present some numerical experiments exploring multiplicity of solutions of (1). Part of the discussion focuses on the well-posedness of these numerical experiments, which are based upon the *Poincaré-Miranda* theorem and its geometric ideas in the context of the *shooting method*. To our knowledge, such numerical approach has been extensively used for single equations, but is lesser-known for systems of ODEs.