Structural stability of global attractors for a gradient ODE with delay

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We study the following gradient ODE

$$x'(t) = f(x(t)) \quad \text{where} \quad f \in C^2(\mathbb{R}^d;\mathbb{R}^d) \quad \text{and} \quad f = \nabla F. \tag{1}$$

Assuming that this equation has a global attractor, we consider the problem perturbed by the delay term

$$x'(t) = f(x(t)) + \varepsilon \int_{-\infty}^{t} M(t-s)x(s)\, ds, \tag{2}$$

where $M : [0,\infty) \to \mathbb{R}^{d\times d}$ is a time dependent matrix whose norm decays to zero exponentially as $t \to \infty$. The main result says, that if in the system (1) the equilibria are hyperbolic and their stable and unstable manifolds intersect transversally, then the structure of heteroclinic connections is preserved in the infinite dimensional system (2) if ε is small. The main tool is the Dafermos transform. We prove that in the perturbed problem the additional infinite dimensional variable that comes from the presence of the delay term after the Dafermos transform is always locally stable. This allows us to demonstrate that the local stable and unstable manifolds of all the equilibria of (2) when ε is small are C^1 close, in appropriate sense, to the local stable and unstable manifolds of (1). Consequently, the structure of transversal intersections is preserved in the perturbed system.