Characterization of compact sets by fixed-point theorems

Maria Japón
University of Sevilla, Spain
japon@us.es

Every fixed-point theorem has two key players: the domain and the operator involved. Some requirements over the topology or geometry of the domain, along with some criteria over the continuity or metric properties of the acting operator, are known that may guarantee the existence of a fixed point.

During this talk, we will focus on domains which are closed convex subsets of a Banach space and we will look at an opposite scope: What can we say about the topological or geometrical features of a domain C when the existence of a fixed point is always guaranteed for some family of Lipschitz operators acting over C? Can we determine any features over the Banach space X in which the domain lives?

Given a topological or metric space C, we say that C has the fixed point property for a given class of mappings $\mathcal{A}$, if every map $T : C \to C$, with $T \in \mathcal{A}$, has a fixed point.

During this talk, we will try to cover a different point of view: Is it possible to determine some topological or geometrical features of the domain C, when it has the fixed point property for a certain class of Lipschitzian mappings?

References

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