

# Partial Blow-up Phenomena in the $SU(3)$ Toda System on Riemann Surfaces

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This talk discusses the partial blow-up phenomena for the  $SU(3)$  Toda system on compact Riemann surfaces with boundary. We consider the following coupled Liouville system with Neumann boundary conditions:

$$\begin{cases} -\Delta_g u_1 = 2\rho_1 \left( \frac{V_1 e^{u_1}}{\int_{\Sigma} V_1 e^{u_1} dv_g} - \frac{1}{|\Sigma|_g} \right) - \rho_2 \left( \frac{V_2 e^{u_2}}{\int_{\Sigma} V_2 e^{u_2} dv_g} - \frac{1}{|\Sigma|_g} \right) & \text{in } \Sigma \\ -\Delta_g u_2 = 2\rho_2 \left( \frac{V_2 e^{u_2}}{\int_{\Sigma} V_2 e^{u_2} dv_g} - \frac{1}{|\Sigma|_g} \right) - \rho_1 \left( \frac{V_1 e^{u_1}}{\int_{\Sigma} V_1 e^{u_1} dv_g} - \frac{1}{|\Sigma|_g} \right) & \text{in } \Sigma \\ \partial_{\nu_g} u_1 = \partial_{\nu_g} u_2 = 0 & \text{on } \partial\Sigma \end{cases},$$

where  $(\Sigma, g)$  is a compact Riemann surface with smooth boundary  $\partial\Sigma$ ,  $\rho_i$  is non-negative parameter and the positive potential function  $V_i$  is smooth for  $i = 1, 2$ .

On a bounded domain the corresponding problem has been studied by D'Aprile, Pistoia, and Ruiz in [1]. On Riemann surface with boundary, We construct a family of blow-up solutions via the Lyapunov-Schmidt reduction and variational methods, where one component remains uniformly bounded from above, and the other exhibits partial blow-up at a prescribed number of points, both in the interior and on the boundary. This construction is based on a non-degeneracy hypothesis for singular mean field equations.

I will discuss constructing solutions for the Toda system with partial blow-ups and analyze the non-degeneracy hypothesis that is important to our approach.

This is a joint work with Prof. Dr. Thomas Bartsch and Prof. Dr. Mohameden Ahmedou.

## References

- [1] Teresa D'Aprile, Angela Pistoia, and David Ruiz. A continuum of solutions for the  $SU(3)$  Toda system exhibiting partial blow-up. *Proc. Lond. Math. Soc.*, 111(4):797–830, 2015.