Realizability theory for partially ordered sets

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In fixed point theory, a sequence (a_n) of non-negative integers is called *realizable* if there exists some space X and a map $T\colon X \to X$ with

$$a_n = \mathrm{Fix}(T^n) = \#\{x \in X \mid T^n x = x\}$$

for all $n \geq 1$.

It is not difficult to observe that a sequence (a_n) is realizable if and only if $b_n = \frac{1}{n}\sum_{d|n} \mu(\frac{n}{d})a_d$ (the so-called Dold coefficient) is a non-negative integer for each $n \geq 1$, where μ denotes the classical Möbius function, cf. [2]. On the other hand, a formal and more abstract theory of Dold coefficients was developed in [1], where instead of the divisibility relation on the integers a partial order on a poset is considered. The aim of this talk is to generalize the notion of realizability to the setting of partially ordered sets and to identify some counterparts of the statements from [3] that are valid in the classical case.

References

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[3] S. Jaidee, J. Byszewski, T. Ward, *Generating all time-changes preserving dynamical zeta functions*, https://doi.org/10.48550/arXiv.2403.13932