Optimal transport techniques in geometric problems

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In the 1980s, Kohn and Strang observed that in the study of some two-dimensional optimal desing problems in plasticity, one can simplify the problem of finding the stress tensor by replacing it with a scalar stress function in such a way that the stress tensor is the rotation by 90 degrees of the gradient of the stress function; then, one needs to consider a constrained minimisation problem for the stress function, with the length of its gradient bounded from above. In recent years, a similar link between the optimal transport problem and the least gradient problem was discovered. On convex domains in two dimensions, given a solution to the optimal transport problem for suitably chosen source and target measures, the gradient of the solution to the least gradient problem (i.e., minimisation of total variation for Dirichlet boundary data) turns out to be a rotation by 90 degrees of the so-called transport density. This provides a one-to-one correspondence of the data and solutions to the respective problems. This greatly increases the number of available tools in the study of the least gradient problem. In this talk we explore this correspondence, some of its generalisations and consequences.