Control Systems and Differential Inclusions on Wasserstein spaces

Hélène Frankowska

CNRS, Institut de Mathématiques de Jussieu - Paris Rive Gauche,
Sorbonne Université, Paris, France
`helene.frankowska@imj-prg.fr`

In recent times models arising in social sciences involving large number of agents are often stated on Wasserstein metric spaces of Borel probability measures. The aim of this talk is to demonstrate that some corner stone results of classical differential inclusions known in the Euclidean framework have their analogues in Wasserstein spaces. Such an extension of the theory of differential inclusions is crucial to study control systems on Wasserstein spaces. Indeed, it is well known that for optimal control of ODEs, the differential inclisions theory provides useful tools to investigate existence of optimal controls, necessary optimality conditions and Hamilton-Jacobi-Bellman equations. Same happens for Wasserstein spaces.

In particular, I will present necessary and sufficient conditions for the existence of solutions to state-constrained continuity inclusions from [2] building on a suitable notion of contingent cones in Wasserstein spaces leading to viability and invariance theorems. In a bit less general setting they were already applied in [4], [5] to investigate stability of controlled continuity equations and uniqueness of solutions to HJB equations.

References

[1] BONNET B. & FRANKOWSKA H., Carathéodory Theory and a Priori Estimates for Continuity Inclusions in the Space of Probability Measures, submitted.

[2] BONNET B. & FRANKOWSKA H., On the Viability and Invariance of Proper Sets under Continuity Inclusions in Wasserstein Spaces, *SIAM Journal on Mathematical Analysis*, to appear.

[3] BONNET B. & FRANKOWSKA H., Differential inclusions in Wasserstein spaces: the Cauchy-Lipschitz framework, *Journal of Diff. Eqs.*, 2021.

[4] BONNET B. & FRANKOWSKA H., Viability and exponentially stable trajectories for differential inclusions in Wasserstein spaces, *Proceedings of 61st IEEE Conference on Decision and Control, Mexico, December 6-9, 2022.*

[5] BADREDDINE Z. & FRANKOWSKA H., Solutions to Hamilton-Jacobi equation on a Wasserstein space, *Calculus of Variations and PDEs*, 2022.