Spectral Analysis of Nonlinear Operators
Old Problems, New Results

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At the beginning of this talk we return back to 1970s when the speaker was a student of S. Fučík and J. Nečas at the Faculty of Mathematics and Physics of Charles University in Prague. During these years topological and variational methods of nonlinear analysis became modern topics which were beginning to be incorporated in the curricula for master programs. Actually, during this period famous Mountain Pass Theorem, Landesman-Lazer Conditions and Rabinowitz Bifurcation Theorem were born and during the same period celebrated papers of Landesman and Lazer, Ambrosetti and Prodi and Ambrosetti and Rabinowitz became were highly cited. Among Czech mathematicians, Svatopluk Fučík was very excited by all these new developments, and since his age was close to that of ours (he was only 9 years older) he succeeded to contaminate us by his enthusiasm. In a short period of time he made a lot of substantial contributions which were published in his numerous papers and books. However, his premature death did not allow him to raise enough continuators of his research. The speaker had the priviledge to be one of few students who were impacted by his personality and this fact determined his future professional career. Particular attention of Fučík and his colleagues was focused on the spectral analysis of nonlinear operators and their contribution to this topic was collected in the monograph of Fučík, Nečas, Souček and Souček, published by Springer Lecture Notes Series in 1973. The ideas to investigate nonlinear problems using asymptotic methods lead to the notions of the Fučík spectrum as well as to the spectrum of the p-Laplacian. Both concepts in ODE setting were studied in speaker's Master Thesis and later also in his PhD Thesis. The fact that this topic was relevant for the future development of nonlinear analysis is illustrated by the vast literature dealing with both of these concepts. Large number of new methods were developed in order to solve several open problems in this field. The goal of this talk is to recall some of the *old problems* which remain open after more than 50 years. On the other hand, we want to present some *new results* which illustrate that nonlinear stucture of the problem might lead to some surprising and unexpected results.