Parabolic systems with cross-diffusion: global existence versus finite time blowup

Piotr Biler

Mathematical Institute, University of Wrocław, Poland

`Piotr.Biler@math.uni.wroc.pl`

Two toy models, both consisting of parabolic systems with nonlinear cross-diffusion terms, obtained after a slight modification of the nonlinearity of the usual doubly parabolic Keller–Segel system

$$u_t = \Delta u - \nabla \cdot (u\nabla\varphi),$$

$$\tau\varphi_t = \Delta\varphi + u,$$

are studied. For these toy models, with the same structure of steady states as is for the nonlinear heat equation $u_t = \Delta u + u^2$, we establish that for data which are, in a suitable sense, smaller than the diffusion parameter τ in the equation for the chemoattractant, we obtain global solutions, and for data larger than τ, a finite time blowup. In this way, we check that our size condition for the global existence is sharp for large τ. Results are based on papers in collaboration with Grzegorz Karch, Dominika Pilarczyk, Hiroshi Wakui and in particular on [1].

References

[1] Piotr Biler, Alexandre Boritchev (Lanar), Lorenzo Brandolese, *Sharp well-posedness and blowup results for parabolic systems of the Keller–Segel type*, Methods and Applications of Analysis **30** (2023), 53–76.