All but one expanding Lorenz maps with slope greater than or equal to $\sqrt{2}$ are leo

Piotr Bartłomiejczyk
Faculty of Applied Physics and Mathematics,
Gdańsk University of Technology,
Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland
piobartl@pg.edu.pl

We prove that, with only one exception, all expanding Lorenz maps with slope greater than or equal to $\sqrt{2}$ are locally eventually onto (leo). To be more precise, recall that an *expanding Lorenz map* is a map $f\colon [0,1] \to [0,1]$ satisfying the following three conditions:

1. there is a critical point $c \in (0,1)$ such that f is continuous and strictly increasing on $[0,c)$ and $[c,1]$,
2. $\lim_{x\to c^-} f(x) = 1$ and $\lim_{x\to c^+} f(x) = f(c) = 0$,
3. f is differentiable for all points not belonging to a finite set $F \subset [0,1]$ and there is $\lambda > 1$ such that $\inf \{f'(x) \mid x \in [0,1] \setminus F\} \geq \lambda$.

Recall also that f is called *locally eventually onto* (*leo* for short) if for every nonempty open set $U \subset [0,1]$ there is $n \in \mathbb{N}$ such that $[0,1] \setminus f^n(U)$ is finite. Assume that f is an expanding Lorenz map and $\beta = \inf \{f'(x) \mid x \in [0,1] \setminus F\}$. Let $f_0(x) = \sqrt{2}x + \frac{2-\sqrt{2}}{2} \pmod 1$. Our main result states that if $\beta \geq \sqrt{2}$ and $f \neq f_0$ then for every nonempty open subinterval $J \subset (0,1)$ there exists $n \in \mathbb{N}$ such that $f^n(J) \supset [0,1)$. In particular, f is leo. This is joint work with Piotr Nowak-Przygodzki.

References

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