Convexity properties of Yoshikawa-Sparr interpolation spaces

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One of the classical problems concerning interpolation theory consists in proving that the interpolation space obtained by a certain method of interpolation inherits properties of one of the spaces from the $(n+1)$-tuple of spaces. First result in this direction was obtained by B. Beauzamy who proved that if one of the spaces from the couple of spaces is uniformly convex then the interpolation space obtained by the Lions-Peetre method with a specific norm also has this property.

In this talk we present a similar theorem for Yoshikawa-Sparr interpolation method, which is a generalization of Lions-Peetre method, however we consider different norms in this space. We also present a method which enables us to obtain results concerning stability of infinite-dimensional counterparts of uniform convexity for the continuous version of Yoshikawa-Sparr interpolation method. In this method the Lebesgue-Bochner space $L_p(X)$ is used, so when we deal with such property as nearly uniform convexity or property (β), the difficulty is that $L_p(X)$ need not have this property even if the space X has it.

However our method enables us to obtain theorems for the continuous method in the easy way, using known results for the discrete one. We also present a new theorem concerning property (β) of a direct sum of Banach spaces, which we use to prove a theorem about interpolation.

The results presented in this talk are contained in the paper [1]

References

[1] Karol Aleksandrowicz, Stanisław Prus, *Convexity properties of Yoshikawa-Sparr interpolation spaces*, Mathematische Nachrichten, 2024.